National Research Lobachevsky State University of Nizhny Novgorod
Admissions and Academic Coordination Office for International Students
603022, Russian Federation, Nizhny Novgorod, Gagarin Avenue, 23
www.unn.ru/eng

DEMO version of Master's degree admission test MATHEMATICS (English medium of instruction)

The admission test of Mathematics (for Master's degree) consists of one part. It includes 5 test questions, which require solving mathematical tasks and answering the given questions. Each test question is worth 20 points. The candidate can get total of 100 points for this part.

The maximum number of points – 100.
The minimum number of points needed to pass the exam successfully – 40.
The candidate has 180 minutes (3 hours) to complete all the tasks.

Part 1

1. Solve the equation $\lim\limits_{x\to 0} \dfrac{x \sin 2x}{1-\cos x}$

2. The linear transformation is defined by the formula

$\varphi(x) = \begin{pmatrix} 2x_1 + x_2 + x_3 \\ ax_1 + x_2 + 2x_3 \\ -3x_1 - x_2 - 2x_3 \end{pmatrix}$, **where** $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

a. At what value of the parameter a number 3 is an eigenvalue of the transformation?
b. Given the value of a, list all the eigennumbers of the transformation in order of non-decreasing multiplicity.
c. For an eigenvalue 3, find an eigenvector with mutually simple integer coordinates in which the first coordinate is positive, and write the sum of its coordinates.
d. At what value of the parameter a is the vector $6, a+7, -6^T$ an eigenvector of the transformation?
e. To which eigenvalue does the eigenvector from (4) correspond?

3. The number of calls to the firm per hour is a random variable X, which has a Poisson distribution with the parameter $\lambda = 4.15$

a. Calculate the probability of the event $P(X = 1)$
b. Calculate the probability of the event $P(|X - 4| < 3)$
c. Calculate the mathematical expectation of the quantity X^2

4. A differential equation is given: $y''(x) - 10y'(x) + 21y(x) = 0$

a. What is the type of this equation?
   - linear homogeneous with constant coefficients
   - linear heterogeneous with variable coefficients
   - with separating variables
   - Cauchy problem
b. Determine the characteristic equation of this equation
   - $\lambda^2 - 10\lambda + 21 = 0$
   - $21\lambda^2 - 10\lambda + 1 = 0$
   - $\lambda^2 + \lambda + 1 = 0$
   - $\lambda = 0$
c. What is the structure of the solution to this equation?
   - $y(x) = C_1 e^{3x} + C_2 e^{7x}$
   - $y(x) = C_1 e^{x} cos3x + C_2 e^{x} sin7x$
   - $y(x) = C_1 e^{3x} sin7x + C_2 e^{7x} cos3x$
   - $y(x) = 3C_1 e^{x} + 7C_2 e^{x}$

5. There are 4 wholesale depots in the city. The probability that the required type of goods is not available at these depots is the same and is equal to 0,24. Draw up the distributive law of the number of bases where the required product is absent at the moment.

Grading scale of Master's degree admission test
MATHEMATICS

Module	Content of module	Number of questions	Number of points for a correct answer to a question	Maximum number of points for a module
1	Multiple-choice test questions / calculus test tasks	5	20	100
IN TOTAL		**-**	**-**	**100**

Topics and Questions for admission test
MATHEMATICS

MATHEMATICAL ANALYSIS

Limits of sequences and functions. First and second remarkable limits and corollaries from them. Equivalent infinitesimal quantities. Disclosure of basic uncertainties, Lopital's rules.
Continuity of functions of one and several variables (at a point, on a set). Cumulative and subordinate continuity. Theorems on continuous functions.
Derivative and differential of a function of one variable. Criterion of differentiability of a function.
Differentiability of a function of several variables. Partial derivatives and differential of a function. Directional derivative.
Necessary and sufficient conditions for the local extremum of a function of several variables. Conditional extremum of a function of several variables. Lagrange's method of indefinite multipliers.
Indefinite integral and its properties. Table of indefinite integrals for elementary functions.
Basic methods of integrating functions (method of substitution of variable, method of integration by parts, method of decomposition into elementary fractions).
The definite integral and its properties.
Calculation of areas of flat areas and volumes of bodies. Length of plane curve in different coordinates.
Basic theorems for curvilinear and surface integrals. Green, Ostrogradsky-Gauss, Stokes formulas.
Degree series. Decomposition of functions into power series. Functional series, Fourier series and integral.

ALGEBRA AND GEOMETRY

Linear vector spaces. Linear independence of systems of vectors. Rank of vector systems. Basis and dimensionality of linear space. Coordinates of a vector in a basis, change of coordinates at change of basis.
Determinant of a matrix, its properties and methods of calculation. Matrix operations. Ring of square matrices. Methods of matrix reversal. Theorem on the rank of the product of matrices.
Systems of linear algebraic equations. Cramer's rule, Gauss method, condition of jointness and solvability. Representation of a general solution in the form of a linear manifold.
Linear transformations of finite-dimensional linear vector space and their matrix representation. Eigenvalues and vectors of linear transformation, methods of their calculation. Matrix similarity relation. Diagonalizable matrices.

Bilinear and quadratic forms and their matrix representation. Canonical representation of a positively defined quadratic form.
Euclidean and unitary spaces. The process of orthogonalization of a system of vectors. Affine and orthogonal classification of curves and surfaces of second order.

DISCRETE MATHEMATICS

Basic concepts of set theory. Algebra of sets. Binary relations and their properties. Equivalence and order relations and their properties.
The notion of power of a set. Basic combinatorial configurations and counting their number. The principle of inclusions-exclusions.
Basic concepts of graph theory. Methods of graph representation. Metric characteristics of graphs. The most important classes of graphs (trees, bipartite graphs, planar graphs). Theorems of Koenig and Pontryagin-Kuratovsky.
Logical functions and ways of their representation. Complete systems of functions. The information theorem. Post's criterion. Precomplete classes.
Statement of the problem of optimal coding. Reduction of the general problem of optimal coding to the problem of constructing an optimal prefix code. Huffman's algorithm.

DIFFERENTIAL EQUATIONS

Differential equations of the first order. Methods of integration of the simplest differential equations (equations with separating variables, homogeneous, linear, Bernoulli, in full differentials and reduced to them, Lagrange and Clero equations).
Cauchy problem. Theorem on existence and uniqueness of the solution of the Cauchy problem. Linear differential equations of higher orders and methods of their solution.
Systems of linear differential equations with constant coefficients and methods of their solution.
Stability of solutions of differential equations. Lyapunov's theorems of stability, instability, stability by first approximation.
Autonomous dynamical systems of the 2nd order. Equilibrium states, their type and character of stability. Phase portrait.